Optically Induced Lensing Effect on a Bose-Einstein Condensate Expanding in a Moving Lattice

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We report the experimental observation of a lensing effect on a Bose-Einstein condensate expanding in a moving 1D optical lattice. The effect of the periodic potential can be described by an effective mass dependent on the condensate quasimomentum. By changing the velocity of the atoms in the frame of the optical lattice, we induce a focusing of the condensate along the lattice direction. The experimental results are compared with the numerical predictions of an effective 1D theoretical model. In addition, a precise band spectroscopy of the system is carried out by looking at the real-space propagation of the atomic wave packet in the optical lattice.

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The experimental realization of Bose-Einstein condensation allowed significant advances in the field of atom optics. Forces resulting from the interaction with coherent light can be used to manipulate coherent matter waves [1]. Bragg scattering from two pulsed laser beams [2] has provided a simple tool for the implementation of atomic mirrors, beam splitters, and diffraction gratings. Superradiant Rayleigh scattering has been used as the gain mechanism for the development of a coherent matter waves amplifier [3]. Atom lasers have been realized, providing pulsed and quasi-cw sources of coherent matter waves [4]. Nonlinearities in the atomic wave equation have been exploited in experiments of nonlinear atom optics, such as the realization of four wave mixing [5] or the observation of soliton propagation [6]. One main difference between atoms and photons is the mass, which can be modified by the presence of a periodic potential, such as that resulting from the interference of two counterpropagating laser beams. The superfluid behavior of a condensate in such an optical lattice has been studied in [7] showing the role of the effective mass in shifting the collective mode frequencies. The possibility of achieving experimental control over the effective mass allows the dispersion management of the matter wave packet. Several fascinating effects are predicted to appear in the negative effective mass regime, such as the formation of gap solitons in a condensate with repulsive interactions [8].

In this Letter, we demonstrate the possibility to change the expansion of a Bose-Einstein condensate (BEC) using a moving optical lattice, which acts as a lens for matter waves, focusing or defocusing the atomic cloud along the direction of the lattice, as recently predicted in [9]. The observed center-of-mass dynamics can be well explained in terms of band structure and Bloch states, familiar concepts to solid state physics. This picture has been confirmed by many experimental results, including the observation of Bloch oscillations [10] and extensive work on loading and manipulating a condensate in an optical lattice [11]. In the rest frame of the lattice, the eigenenergies of the system are $E_n(q)$, where $q$ is the quasi-momentum and $n$ the band index. According to band theory, the velocity in the $n$th band is $v_n = \hbar^{-1} \partial E_n / \partial q$ and the effective mass is $m^* = \hbar^2 (\partial^2 E_n / \partial q^2)^{-1}$. We demonstrate that the expansion of the condensate is strongly modified by the change in the single particle effective mass $m^*$, which enters the diffusive (kinetic) term in the Gross-Pitaevskii equation. The expansion of a BEC in a static optical lattice has been already studied in [12]. Here we use a moving optical lattice to load the condensate in quasimomentum states with different effective mass. This ability to access regions of negative effective mass allows us to change the sign of the matter wave dispersion, inducing the condensate to compress along the lattice direction instead of expanding [9].

The experiment is performed on a Bose-Einstein condensate of $^{87}\text{Rb}$ produced in a standard double magneto-optical trap apparatus by means of combined laser and rf-evaporative cooling. The evaporation is performed in a Ioffe-Pritchard magnetic trap with frequencies $\nu_\perp = 8.8$ Hz and $\nu_\parallel = 90$ Hz along the axial and radial directions, respectively. We typically produce condensates of $\approx 10^5$ atoms in the hyperfine level $|F = 1, m_F = -1 \rangle$ of the ground state. The optical lattice is provided by two counterpropagating phase-locked laser beams aligned along the axial direction of the cigar-shaped condensate. The two beams are circularly polarized and blue-detuned 0.5 nm from the Rb D2 line at $\lambda = 780$ nm. The interference of the two beams, derived by the same Ti:Sa laser and controlled by two independent acousto-optic modulators, produces an optical lattice moving at velocity $v_L = \lambda \Delta \nu / 2$, where $\Delta \nu$ is the frequency difference of the two beams. In the laboratory frame, the resulting potential can be written as $V = s E_k \cos^2[k(x - v_L t)]$, where $s$ is the lattice spacing, $E_k$ the kinetic energy of an atom, $k$ the wave vector of the laser, and $x$ and $t$ the spatial and temporal coordinates, respectively.
where \( k = 2\pi/\lambda \) is the modulus of the wave vector and \( s \) measures the depth of the optical lattice in units of the recoil energy \( E_R = \hbar k^2/2m \).

The experiment is performed as follows (see Fig. 1). After producing the BEC, we switch off the magnetic trap and let the atomic cloud expand. After 1 ms of expansion, we adiabatically switch on the moving lattice by ramping the intensity of the two laser beams in 2 ms. We let the condensate expand in the lattice and, after a total expansion time of 13 ms, we take an absorption image of the cloud along the radial horizontal direction looking at the position and dimensions of the condensate. We note that the waist of the laser beams (about 2.0 mm) is big enough to provide a constant light intensity during the entire expansion of the condensate. This loading procedure allows us to project the condensate in a Bloch state of well-defined energy and quasimomentum [11]. We verified the adiabaticity of this procedure by checking that, applying the reverse ramp to switch off the lattice, at the end of the expansion we still have only one momentum component in the atomic cloud (i.e., we have populated only one energy band). In our experiments, we typically move the optical lattice with velocities \( v_L \) between 0 and \( 2v_B \), where \( v_B = q_B/m = \hbar k/m = 5.89 \text{ mm/s} \) is the recoil velocity of an atom absorbing one lattice photon. As a result of the adiabatic loading, we can access different energy bands: for \( 0 < v_L < v_B \) we populate the first band, while for \( v_B < v_L < 2v_B \) the second band is populated.

From the measured positions of the condensate center of mass at the end of the expansion, we extract the velocity of the atoms inside the periodic potential. In the moving frame of the lattice, the atomic velocity is given by \( v = v_L - \Delta x/\Delta t \), where \( \Delta x \) is the axial displacement of the condensate and \( \Delta t \) is the time of expansion inside the optical lattice (Fig. 1). In Fig. 2(a), we report the experimental velocities as a function of quasi-momentum \( q/q_B \) for two different lattice depths: \( s = 1.3(1) \) and \( s = 3.8(1) \). The error bars include the indeterminacy in \( \Delta t \) due to the adiabatic switching on of the optical lattice. The lines shown in the figure are obtained from the calculation of the velocity in the first two energy bands. The measured spectrum of velocities shows very good agreement with theory. We note that the theoretical curves are derived from the simple one-particle model neglecting the effect of interactions. As a matter of fact, since the experiment is performed after some expansion, we expect that interactions play a negligible role on the energy spectrum (after 2 ms of expansion the interaction energy has been almost completely converted into kinetic energy). An adequate sampling of the experimental velocities allows us to reconstruct the effective mass by evaluating the derivative \( \partial v/\partial q \) from the finite increment between consecutive points. In Fig. 2(b), we report the results of such an analysis on the data taken at \( s = 1.3 \) together with the theoretical curve. This experimental study allows us to make a precise spectroscopy of the energy bands, measuring the velocity spectrum and the effective mass of the condensate in the periodic potential.

![FIG. 1. Schematics of the experimental procedure. After releasing the condensate from the magnetic trap (A), we adiabatically ramp the intensity of an optical lattice moving at velocity \( v_L \). We let the condensate expand in the periodic potential and, after 10 ms at the maximum light intensity, we look at the position and dimensions of the atomic cloud by absorption imaging along the radial horizontal direction (B).](image)

![FIG. 2. (a) Velocity of the condensate in the frame of the moving lattice for the lowest two energy bands and two different optical intensities: \( s = 1.3 \) (closed circles) and \( s = 3.8 \) (open circles). The experimental data are obtained from the measured displacements of the condensate center of mass after the expansion inside the lattice. The lines are calculated from band theory. (b) Effective mass of the condensate in the lowest two energy bands for \( s = 1.3 \). The experimental points (closed circles) are obtained by numerically evaluating the incremental ratios \( \Delta v/\Delta q \) from the data shown above. The lines are calculated from band theory. We remember that \( v_B = q_B/m = \hbar k/m \).](image)
dimensions of the expanded condensate on different lattice velocities (hence, different quasimomenta of the condensate in the frame of the moving lattice). Typical absorption images for different $q$ are reported in the upper part of Fig. 3. In the bottom of Fig. 3, we report the measured Thomas-Fermi radii of the condensate as obtained from a 2D fit of the measured density distribution [13]. We note that, approaching the boundary of the first Brillouin zone for $q \approx q_B$, the axial dimension of the condensate gets smaller as a consequence of the modified effective mass $m^* < 0$ [see Fig. 3(b)]. In fact, it is easy to show that a change of sign in the effective mass corresponds to a time-reversed evolution under the influence of an inverted external potential (if present). Since in our case the condensate is initially expanding outwards, when $m^*$ becomes negative an inversion of dynamics takes place [14]. This contraction continues for times much longer than those considered in this experiment, until the wave packet eventually reaches its minimum allowed size (when dynamics inverts again).

This focusing effect along the axial direction is balanced by an increased expansion along the radial axis, which we attribute to an effect of interactions. In fact, due to the compression along the lattice direction, the fast radial expansion is further enhanced by the increase of the mean-field energy. When the condensate is loaded in the second band, for $q \approx q_B$, the axial expansion is enhanced due to the strong positive curvature of the second energy band near the zone boundary, where $0 < m^* < m$ [see Fig. 3(c)]. As one would expect, in this case the radial dynamics is not modified, since the residual mean-field energy is further reduced by the fast axial expansion, causing a suppression of the nonlinear coupling between the axial and radial dynamics.

To get further insight on the behavior of the condensate during the expansion, we have analyzed the experimental results by means of the 1D effective model presented in [9]. According to this model, the full 3D Gross-Pitaevskii description of the system is first dynamically rescaled by using the unitary scaling and gauge transformations of [16,17], and then reduced to an effective 1D equation (dynamically rescaled Gross-Pitaevskii equation) by a Gaussian factorization of the radial wave function [18]. Despite its 1D nature, the model is capable to account both for the axial and radial dynamics of the system, as discussed in [9]. In the present case, the factorization of the wave function is further justified by the fact that during the expansion the axial and radial degrees of freedom almost decouple.

Actually, Fig. 3 shows that the model qualitatively reproduces the behavior observed in the experiment, even though it does not fit precisely the data. In particular, approaching the first zone boundary, the observed focusing effect along the axial direction is slightly smaller.

![Figure 3](image3.png)

**FIG. 3.** Absorption images of the expanded condensate. From left to right: (a) normal expansion of the condensate without lattice; (b) axial compression in a lattice with $s = 2.9$ and $v_L = 0.9v_B$; (c) enhanced axial expansion in a lattice with $s = 2.9$ and $v_L = 1.1v_B$ (where $v_B = q_B/m = \hbar k/m$). In the lower part, we report the axial and radial dimensions of the condensate after expansion in an optical lattice with $s = 2.9$ as a function of the quasimomentum. The experimental points (closed and open circles) show the Thomas-Fermi radii of the cloud extracted from a 2D fit of the density distribution. The dotted lines show the dimensions of the expanded condensate in the absence of the optical lattice. The continuous and dashed lines are theoretical calculations obtained from the 1D effective model described in the text.

![Figure 4](image4.png)

**FIG. 4.** Ratio of the radial to the axial size (aspect ratio) of the condensate after 13 ms of expansion in an optical lattice with $s = 2.9$. The dotted line shows the aspect ratio of the expanded condensate in the absence of the optical lattice. The continuous line is a theoretical calculation obtained from the 1D effective model described in the text.
than the calculated one, and at the same time the expansion along the radial direction (not directly affected by the lattice) is enhanced. Instead, in the second band the radial behavior is well reproduced by the model, whereas there is still a discrepancy concerning the axial expansion. We remark that in the region near to the band edge ($0.95q_B < q < 1.05q_B$) the process of switching on the optical lattice is no longer adiabatic, and the description is complicated by the fact that more than one energy band gets populated. Indeed, what we actually see in the experiment is a superposition of two atomic clouds with different shapes resulting from the minor population of a different energy band. At the present stage of the experiment, we cannot increase arbitrarily the ramp time of the lattice intensity as the condensate, under the effect of gravity, falls out of the lattice beams.

In conclusion, we have achieved a lensing effect on a Bose-Einstein condensate expanding inside a moving optical lattice. Tuning the velocity of the lattice, we can set the lensing power of the periodic potential, all the way from focusing to defocusing of the atomic cloud. The demonstrated experimental control of the matter wave dispersion is likely to open new possibilities in the field of atom optics, including the observation of nonlinear effects such as the generation of gap solitons. The experimental techniques introduced in this work will also allow future high precision studies of the band structure of a Bose-Einstein condensate in an optical lattice.

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Note added.—After completion of this work, we became aware of the publication of a paper [19] showing results similar to those presented in this Letter.

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[13] Because of fluctuations ($\sim 20\%$) in the number of atoms in the different experiments, we scaled the measured radii to the average number of atoms using the relation $R \approx N^{1/3}$ valid in the Thomas-Fermi limit. Each data point in Fig. 3 corresponds to an average of ten repeated measurements.

[14] Note that, if the sign of the effective mass $m^*$ changes when the condensate is still at rest, after the release from the trap it would expand regardless of the sign of $m^*$.

[15] We note that, while the dimensions of the condensate really depend on the number of atoms, in the Thomas-Fermi limit the aspect ratio is not affected by fluctuations in the size of the condensate.


